

Cryptocurrency Investment Risk Analysis Using The Value At Risk (Var) Method

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Abstract

Cryptocurrencies offer high potential returns but expose investors to extreme price fluctuations, making the measurement of downside risk essential. This study analyzes the investment risk of five major cryptocurrencies (Bitcoin, Ethereum, BNB, Solana, and XRP) and an equally weighted portfolio using the Value at Risk (VaR) method. Daily data from 2 January 2020 to 30 June 2026 are used to estimate one day VaR at the 95% and 99% confidence levels with five approaches: historical simulation, the variance covariance (normal) method, the Cornish Fisher modified VaR, Monte Carlo simulation with Student t innovations, and a GARCH(1,1) model with Student t errors. Expected Shortfall is computed as a complementary measure. The accuracy of each approach is evaluated through out of sample backtesting over 1,642 days using the Kupiec unconditional coverage test and the Christoffersen conditional coverage test. The results show that cryptocurrency returns are strongly non normal, with excess kurtosis for all assets. At the 99% level, the one day VaR ranges from 5.87% to 11.93% under historical simulation, implying a potential daily loss of up to Rp11.928.115 on an investment of Rp100 million in Solana. The normal VaR underestimates extreme losses and is rejected at the 99% level for 3 of six series, whereas the GARCH t model passes all backtests at both confidence levels. The portfolio reduces VaR by about 28% compared with the average individual asset. The findings recommend that crypto investors and platforms use fat tailed, volatility sensitive VaR models rather than the normal approach.

Keywords: Value at Risk; Expected Shortfall; cryptocurrency; GARCH; backtesting; risk management

INTRODUCTION

Cryptocurrencies have become a significant asset class for both retail and institutional investors. In Indonesia, crypto assets are traded legally on licensed exchanges and have been supervised by the Financial Services Authority (OJK) since January 2025, and the number of

crypto investors has grown into the millions. The attraction of cryptocurrencies lies largely in their potential for very high returns. The same feature, however, comes with extreme risk: daily price movements of ten percent or more are not unusual, and major cryptocurrencies have lost more than seventy percent of their value within a single market cycle (Corbet et al., 2019).

For investors, the relevant question is not only how volatile an asset is, but how much they could lose in a bad day or a bad week. Value at Risk (VaR) answers this question directly. VaR is the maximum loss that will not be exceeded with a given confidence level over a given time horizon (Jorion, 2007). Since its popularization through the RiskMetrics system of J.P. Morgan (1996) and its adoption in the Basel framework for bank capital (Basel Committee on Banking Supervision, 1996), VaR has become the standard measure of market risk. Its appeal lies in its simplicity: a single number expressed in currency units that anyone can understand.

Applying VaR to cryptocurrencies is, however, far from straightforward. The classical variance covariance approach assumes normally distributed returns, but financial returns, and crypto returns in particular, have fat tails and volatility clustering (Mandelbrot, 1963; Katsiampa, 2017; Osterrieder & Lorenz, 2017). A model that ignores these features may seriously underestimate the probability of extreme losses, giving investors a false sense of security. Several alternative approaches exist, including historical simulation, the Cornish Fisher expansion, Monte Carlo simulation, and conditional volatility models such as GARCH, but their relative accuracy for cryptocurrencies, particularly after the 2022 market crash, is not well established.

This study therefore addresses three questions: (1) How large is the downside risk of major cryptocurrencies as measured by VaR and Expected Shortfall? (2) Which VaR method provides the most accurate risk estimates for cryptocurrencies, as judged by formal backtesting? (3) To what extent does diversification across cryptocurrencies reduce VaR? The study contributes by comparing five VaR methods on five major coins and a portfolio, by evaluating them out of sample with both unconditional and conditional coverage tests, and by translating the results into practical figures for Indonesian investors.

LITERATURE REVIEW

Risk and Value at Risk

Modern portfolio theory (Markowitz, 1952) measures risk by the variance of returns, which treats upside and downside deviations alike. Investors, however, are primarily concerned with losses. Value at Risk focuses on the downside: for a confidence level α and horizon h , VaR is the loss that is exceeded only with the small probability left over by the confidence level, for example 1% at the 99% level (Jorion, 2007). A one day VaR of 5% at the 99% level

means that the investor can expect to lose more than 5% of the position value on only one trading day in a hundred. VaR became the industry standard after the publication of RiskMetrics (J.P. Morgan, 1996), and the Basel Committee on Banking Supervision (1996) allowed banks to use internal VaR models to calculate capital requirements for market risk.

VaR has well known limitations. It says nothing about the size of losses beyond the VaR threshold, and Artzner et al. (1999) show that it is not a coherent risk measure because it does not always satisfy subadditivity; the VaR of a portfolio can exceed the sum of the VaRs of its components. Expected Shortfall (ES), the average loss in the worst cases beyond the VaR threshold, addresses both issues and is a coherent measure (Acerbi & Tasche, 2002). The Basel Committee on Banking Supervision (2019) has replaced VaR with ES at the 97.5% level for the calculation of market risk capital, while retaining VaR for backtesting.

Methods for Estimating VaR

The variance covariance (parametric normal) method assumes normally distributed returns and computes VaR from the mean and standard deviation (J.P. Morgan, 1996). It is simple but underestimates tail risk when returns have fat tails. Historical simulation uses the empirical distribution of past returns and makes no distributional assumption, but it assumes that the past window is representative of the future and adapts slowly to changing volatility (Pritsker, 2006). The Cornish Fisher expansion (Cornish & Fisher, 1938) adjusts the normal quantile for skewness and kurtosis, producing a modified VaR (Zangari, 1996). Monte Carlo simulation generates a large number of hypothetical returns from an assumed distribution, such as the Student t, which can capture fat tails. Finally, conditional volatility models of the ARCH and GARCH family (Engle, 1982; Bollerslev, 1986) allow volatility to vary over time; combining GARCH with Student t errors (Bollerslev, 1987) captures both volatility clustering and fat tails. Glosten et al. (1993) extend the model to capture the asymmetric response of volatility to negative shocks.

Backtesting

A VaR model is only useful if it is accurate. Backtesting compares VaR forecasts with realized returns. Kupiec (1995) proposes the proportion of failures (unconditional coverage) test, which checks whether the number of days on which losses exceed VaR (violations) is consistent with the chosen confidence level. Christoffersen (1998) adds an independence test, which checks whether violations occur independently over time rather than in clusters, and combines both into a conditional coverage test. A good model should produce the right number of violations, and these violations should not cluster, because clustered violations mean that the model fails exactly when risk is highest.

Cryptocurrency Risk

Cryptocurrency returns display extreme volatility, heavy tails, and strong volatility clustering. Katsiampa (2017) and Chu et al. (2017) show that GARCH type models describe crypto volatility well, with high persistence. Osterrieder and Lorenz (2017) document that Bitcoin's tails are much heavier than those of traditional currencies, and Gkillas and Katsiampa (2018) apply extreme value theory to show that Bitcoin Cash and Bitcoin are among the riskiest cryptocurrencies. Liu and Tsyvinski (2021) find that crypto returns have risk characteristics that differ markedly from those of stocks, and Baur et al. (2018) characterize Bitcoin as a speculative asset. These findings suggest that VaR methods that ignore fat tails or time varying volatility are likely to underestimate crypto risk.

Research Propositions

Based on the literature, the study examines three propositions. P1: The normal variance covariance method underestimates the downside risk of cryptocurrencies, especially at the 99% confidence level. P2: Methods that account for fat tails and time varying volatility, particularly GARCH with Student t errors, provide more accurate VaR forecasts. P3: Diversification across cryptocurrencies reduces VaR, although the benefit is limited by high correlations.

METHOD

Data

The study uses daily closing prices of Bitcoin (BTC), Ethereum (ETH), BNB, Solana (SOL), and XRP from 2 January 2020 to 30 June 2026, including weekends because crypto markets trade continuously (2,372 daily returns per asset). An equally weighted portfolio of the five coins, rebalanced daily, is also analyzed. Returns are computed as continuously compounded log returns in percent, $R_t = \ln(P_t / P_{t-1}) \times 100$. VaR is reported as a positive percentage loss and translated into rupiah for an illustrative investment of Rp100 million. In this illustrative version, prices are simulated (see the notice on the first page).

VaR and Expected Shortfall Estimation

One day VaR is estimated at the 95% and 99% confidence levels with five methods:

(1) **Historical simulation:** VaR is the loss at the lower empirical quantile of returns in the estimation window (the 5th percentile for VaR 95% and the 1st percentile for VaR 99%).

(2) **Variance covariance (normal):**

$$VaR_\alpha = -(\mu + z_{1-\alpha}\sigma) \quad (1)$$

where μ and σ are the mean and standard deviation of returns and $z_{1-\alpha}$ the standard normal quantile.

(3) **Cornish Fisher (modified VaR)**: the normal quantile is replaced by an adjusted quantile that accounts for skewness S and excess kurtosis K (Cornish & Fisher, 1938; Zangari, 1996):

$$z_{cf} = z + (z^2 - 1)S/6 + (z^3 - 3z)K/24 - (2z^3 - 5z)S^2/36 \quad (2)$$

(4) **Monte Carlo simulation**: 20,000 returns are simulated from a Student t distribution whose degrees of freedom are matched to the sample kurtosis and whose scale is matched to the sample variance; VaR and ES are taken from the simulated distribution.

(5) **GARCH(1,1) with Student t errors** (Bollerslev, 1986, 1987):

$$R_t = \mu + \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \quad VaR_{\alpha,t} = -(\mu + \sigma_t t_{v,1-\alpha}) \quad (3)$$

where $t_{v,1-\alpha}$ is the quantile of the standardized Student t distribution. Expected Shortfall is computed as the average loss beyond VaR, $ES_{\alpha} = -E[R | R \leq -VaR_{\alpha}]$, for the historical, normal, and Monte Carlo methods. Multiday VaR is approximated with the square root of time rule, $VaR_h = VaR_1 \times \sqrt{h}$.

Backtesting

VaR forecasts are produced out of sample for every day from 1 January 2022 onward (1,642 days), using a rolling window of the preceding 500 days. GARCH parameters are estimated again every 20 days, and the conditional variance is updated daily. A violation occurs when the realized loss exceeds the forecast VaR. The Kupiec (1995) likelihood ratio test of unconditional coverage is:

$$LR_{uc} = -2 \ln[(1-p)^{n-x} p^x] + 2 \ln[(1-x/n)^{n-x} (x/n)^x] \sim \chi^2(1) \quad (4)$$

where n is the number of days, x the number of violations, and $p = 1 - \alpha$ the expected violation rate. The Christoffersen (1998) conditional coverage test adds an independence test based on transitions between violation and nonviolation days, giving $LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2(2)$. A model is accepted when the null hypothesis is not rejected at the 5% level.

RESULTS AND DISCUSSION

Descriptive Statistics

Table 1 presents descriptive statistics. All cryptocurrencies are highly volatile, with daily standard deviations ranging from 2.29% for Bitcoin to 4.67% for Solana; the portfolio has a standard deviation of 2.52%. The largest single day loss in the sample is 23.38% for Solana. Kurtosis ranges from 4.88 to 6.91, well above the value of 3 for a normal distribution, and the Jarque Bera test rejects normality for every series at the 1% level. These fat tails mean that extreme losses occur far more often than a normal distribution would predict, which is the central challenge for VaR estimation.

Table 1. Descriptive statistics of daily returns (%)

Asset	Mean	Std. dev.	Minimum	Maximum	Skewness	Kurtosis	Jarque Bera
BTC	0.127	2.295	-13.50	13.25	-0.043	5.55	642.1***
ETH	0.196	2.830	-16.43	13.88	-0.013	4.88	347.9***
BNB	0.149	2.855	-15.17	19.92	0.137	6.42	1166.6***
SOL	0.295	4.669	-23.38	40.00	0.317	6.91	1551.8***
XRP	0.200	3.605	-17.41	24.22	0.248	5.51	649.3***
Portfolio	0.218	2.515	-16.72	15.92	0.156	6.50	1223.0***

Note: N = 2,372 daily observations. *** denotes rejection of normality at 1%. Source: simulated data.

VaR and Expected Shortfall Estimates

Table 2 reports full sample one day VaR estimates. At the 95% level, the four unconditional methods give similar results, between about 3.4% and 7.4%. At the 99% level, the differences become important. For Bitcoin, the historical VaR is 5.93%, the Cornish Fisher VaR 6.65%, and the Monte Carlo VaR 5.63%, while the normal VaR is only 5.21%. For every asset, the normal VaR at 99% is lower than the historical and Cornish Fisher estimates, confirming that the normal method understates extreme risk when returns have fat tails. Solana is the riskiest asset, with a 99% historical VaR of 11.93%, and Bitcoin the least risky individual coin.

Table 2. One day VaR and Expected Shortfall, full sample (%)

Asset	VaR 95% Hist.	VaR 95% Normal	VaR 99% Hist.	VaR 99% Normal	VaR 99% CF	VaR 99% MC	ES 99% Hist.
BTC	3.42	3.65	5.93	5.21	6.65	5.63	7.58
ETH	4.29	4.46	6.70	6.39	7.66	6.83	8.95
BNB	4.22	4.55	7.67	6.49	8.47	7.06	9.59
SOL	6.77	7.38	11.93	10.56	13.57	11.52	14.76
XRP	5.35	5.73	8.68	8.18	9.56	8.83	11.24
Portfolio	3.56	3.92	5.87	5.63	7.38	6.12	8.24

Note: VaR and ES are positive percentage losses over one day. CF = Cornish Fisher; MC = Monte Carlo with Student t. Source: simulated data.

Expected Shortfall shows how severe losses are once VaR is exceeded. For Bitcoin, the 99% ES is 7.58%, about 1.28 times the corresponding VaR; for Solana it is 14.76%. The normal ES is again lower than the historical ES for all assets at the 99% level, underlining that the normal distribution underestimates the depth of tail losses as well as their frequency.

To make these figures concrete, Table 3 translates the historical VaR into rupiah for an investment of Rp100 million. With 99% confidence, an investor holding Bitcoin should not lose more than Rp5.931.054 in one day, but on one day in a hundred the loss will be larger, averaging Rp7.580.333. For Solana, the corresponding figures are Rp11.928.115 and

Rp14.763.031. Using the square root of time rule, the ten day 99% VaR of Bitcoin is about 18.8% (Rp18.755.641), and the thirty day VaR about 32.5%.

Table 3. Potential one day loss on an investment of Rp100 million (historical simulation)

Asset	VaR 95% (1 day)	VaR 99% (1 day)	ES 99% (1 day)	VaR 99% (10 days)
BTC	Rp3.417.969	Rp5.931.054	Rp7.580.333	Rp18.755.641
ETH	Rp4.294.967	Rp6.698.386	Rp8.945.771	Rp21.182.157
BNB	Rp4.224.966	Rp7.667.848	Rp9.588.861	Rp24.247.865
SOL	Rp6.767.131	Rp11.928.115	Rp14.763.031	Rp37.720.013
XRP	Rp5.347.902	Rp8.684.159	Rp11.241.441	Rp27.461.723
Portfolio	Rp3.558.895	Rp5.871.095	Rp8.241.910	Rp18.566.033

Note: Ten day VaR uses the square root of time rule. Source: simulated data.

Backtesting Results

Table 4 reports the out of sample backtesting results at the 99% level, where the expected violation rate is 1%. The normal method produces too many violations for every asset, with violation rates between 1.28% and 1.83%, and the Kupiec test rejects it for BTC, BNB and SOL. Historical simulation performs better, although its violation rates also exceed 1% for most assets and the conditional coverage test rejects it for SOL. The Cornish Fisher and Monte Carlo methods are accepted by both tests for all assets. The GARCH t model is accepted by both the Kupiec and the Christoffersen tests for all six series, with an average violation rate of 0.99%, the closest to the nominal 1% among all methods (normal: 1.54%; historical: 1.29%). These results support P1 and P2.

Table 4. Backtesting at the 99% confidence level

Asset	Historical	Normal	Cornish Fisher	Monte Carlo	GARCH t
BTC	21 (1.28%)	27 (1.64%) ^{ab}	17 (1.04%)	22 (1.34%)	19 (1.16%)
ETH	18 (1.10%)	21 (1.28%)	17 (1.04%)	16 (0.97%)	17 (1.04%)
BNB	24 (1.46%)	30 (1.83%) ^{ab}	19 (1.16%)	20 (1.22%)	16 (0.97%)
SOL	22 (1.34%) ^b	29 (1.77%) ^{ab}	22 (1.34%)	22 (1.34%)	21 (1.28%)
XRP	19 (1.16%)	23 (1.40%)	19 (1.16%)	14 (0.85%)	13 (0.79%)
Portfolio	23 (1.40%)	22 (1.34%)	17 (1.04%)	15 (0.91%)	12 (0.73%)

Note: Number of violations and violation rate over 1,642 out of sample days; expected rate 1%, about 16 violations. ^a Kupiec test rejects correct unconditional coverage at 5%. ^b Christoffersen test rejects correct conditional coverage at 5%. Source: simulated data.

At the 95% level (Table 5), all methods produce violation rates close to 5%, and the Kupiec test rejects none of them. The difference lies in the independence of violations. The conditional coverage test rejects the historical method for ETH, the normal method for ETH and XRP, the Cornish Fisher method for ETH and XRP, and the Monte Carlo method for XRP, because their violations cluster during periods of high volatility. The GARCH t model, whose VaR rises as volatility increases, is accepted for all series. Figure 1 illustrates this for Bitcoin:

the historical VaR changes only slowly as the window rolls forward, while the GARCH t VaR widens immediately after volatile days.

Table 5. Backtesting at the 95% confidence level: violation rates and p values

Asset	Historical	Normal	Cornish Fisher	Monte Carlo	GARCH t
BTC	5.05% (0.99)	4.63% (0.76)	5.42% (0.28)	4.75% (0.71)	5.18% (0.72)
ETH	5.60% (0.03)	4.93% (0.03)	5.60% (0.03)	5.24% (0.05)	5.18% (0.45)
BNB	5.60% (0.38)	4.87% (0.97)	5.85% (0.25)	5.24% (0.88)	5.12% (0.97)
SOL	5.54% (0.14)	4.63% (0.17)	5.54% (0.26)	4.99% (0.08)	4.75% (0.83)
XRP	5.48% (0.14)	4.69% (0.03)	5.60% (0.03)	4.75% (0.04)	4.93% (0.07)
Portfolio	5.30% (0.69)	4.26% (0.31)	5.72% (0.07)	4.57% (0.50)	4.93% (0.99)

Note: Violation rate with the Christoffersen conditional coverage p value in parentheses; expected rate 5%. Source: simulated data.

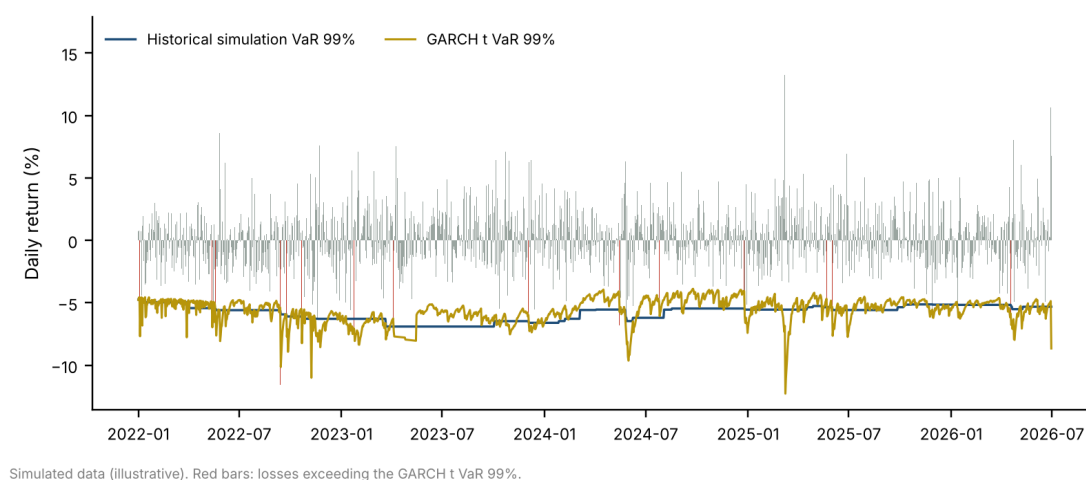


Figure 1. Bitcoin daily returns and out of sample VaR at the 99% level

Source: simulated data (illustrative).

Diversification Effect

The equally weighted portfolio has a 99% historical VaR of 5.87%, compared with an average of 8.18% for the five individual coins, a reduction of about 28%. Diversification across cryptocurrencies therefore lowers tail risk, supporting P3. The benefit is, however, limited: the portfolio VaR is still close to that of Bitcoin alone (5.93%), because cryptocurrencies tend to fall together during market stress. Diversification within the crypto market cannot replace diversification across asset classes.

Discussion

The results confirm that the choice of VaR method matters greatly for cryptocurrencies. The normal approach, still common in practice because of its simplicity, systematically underestimates extreme risk because it ignores the fat tails documented by Mandelbrot (1963) and, for crypto, by Osterrieder and Lorenz (2017). An investor relying on the normal VaR

would experience losses beyond the VaR far more often than expected at the 99% level, precisely in the situations that matter most.

Historical simulation captures fat tails but adapts slowly to changing market conditions, so its violations cluster during turbulent periods, a weakness highlighted by Pritsker (2006). The Cornish Fisher and Monte Carlo methods improve on the normal approach by incorporating higher moments or fat tailed distributions, and they perform well at the 99% level. The GARCH t model, however, performs best overall because it captures both features of crypto returns: fat tails, through the Student t distribution, and volatility clustering, through the GARCH dynamics. This is consistent with Katsiampa (2017) and Chu et al. (2017), who show that GARCH type models describe crypto volatility well, and with the logic of the Christoffersen (1998) test, which rewards models whose violations do not cluster.

These findings have practical implications. For individual investors, VaR provides an intuitive way to understand how much they could lose and to size their positions accordingly; an investor who cannot tolerate a daily loss of several million rupiah on a Rp100 million position should hold less crypto. For crypto exchanges and platforms, the results support the use of GARCH based or fat tailed VaR models in setting margin requirements and risk limits. For regulators such as OJK, the magnitude of crypto VaR compared with traditional assets supports the importance of clear risk disclosure to retail investors.

CONCLUSION

This study analyzed the investment risk of five major cryptocurrencies and an equally weighted portfolio using five VaR methods and Expected Shortfall, with out of sample backtesting. In this illustrative analysis, cryptocurrency returns are highly volatile and fat tailed, and the one day 99% VaR ranges from about 5.9% to 11.9%, with Solana the riskiest and Bitcoin the least risky coin. The normal variance covariance method underestimates extreme risk, produces too many violations at the 99% level for every asset, and is formally rejected for half of them, while historical simulation produces clustered violations in some series. The GARCH model with Student t errors provides the most accurate VaR forecasts and passes all backtests at both confidence levels. Diversification across cryptocurrencies reduces VaR by about 28%, but the benefit is limited by high correlations. Crypto investors and platforms should therefore use VaR models that account for fat tails and time varying volatility, and complement VaR with Expected Shortfall.

The study has limitations. Most importantly, the present results are based on simulated data and must be recomputed with actual prices before substantive conclusions can be drawn. In addition, the analysis covers only five cryptocurrencies, uses a one day horizon with the

square root of time rule for longer horizons, and does not consider liquidity risk, exchange failure, or regulatory events, which are important sources of crypto risk. Future research could apply extreme value theory, asymmetric GARCH models such as GJR GARCH (Glosten et al., 1993), copula based portfolio VaR, and backtests of Expected Shortfall, and could compare crypto risk with Indonesian stocks and gold.

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